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SOME CONTRIBUTION TO OPERATOR THEORY

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This paper is related to operator theory and an important theorem has been discussed.

INTRODUCTION

We have been studying operator theory since a long time. Hilbert space is a special case of Banach space with the added structure of inner product. A continuous linear transformation from a Hilbert space H to H is called an operator. There are different types of operators such as symmetric operator, self adjoint operator, normal operator, nuclear operator and others (Chicago, 1980; Friedrichs, 1973; Bachman and Narici, 1967). We use this additional structure to obtain a deeper insight into the nature of spectra of operators and their perturbations. We also introduce the concept of symmetric and self adjoint operator and study criteria for self adjointness.

Theorem : The numerical range of any operator on x is a convex subset of the complex plane. If K denotes the closure of the numerical range, then either $\not\subset$ /K is connected or K is a strip enclosed by two parallel straight lines.

Proof : Let T be an operator on X and let u, v be unit vectors in D(T) such that $< Tu, u >= \alpha$, $< Tv, v>= \beta$. In order to prove the convexity of η (T) we have to show that for any $0 < P < 1$ there exists a unit vector w in D(T) such that $p\alpha + (l\text{-}p)\ \beta = <$ Tw, w>. Since η (aT+b) = a η (T)+b we may assume without loss of generality that $\alpha = 1$, $\beta = 0$. Further we can change u, v to $e^i\theta$ u, $e^i\phi$ v respectively, where $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$ and hence we may assume without loss of generality that

$$\text{Im}[<Tu,v> + < Tv,u >] = 0$$

Let

$$w = \frac{u+tv}{\|u+tv\|}$$

We shall choose t > 0 suitably so that w becomes the required unit vector. We have

$$< Tw,w > = \frac{1+2t\gamma}{1+t^2+2t\delta}$$

Where

$$2\gamma = \text{Re}[<Tu,\ v> + < Tv,\ u>]$$

$$\delta = \text{Re} < u,\ v >$$

Then w will satisfy our requirements if we can find t such that

$$p = \frac{1+2\gamma\, t}{1+t^2+2t\delta}$$

Solving this quadratic equation for t, we obtain

$$t = p^{-1}\left\{(\gamma\text{-}\ p\delta)+[(\gamma\text{-}\ p\delta)^2 + p(1\text{-}\ p)]^{1/2}\right\} > 0.$$

Hence a w with the required property exists and the first part is proved.

To prove the second part, assume that $\not\subset$ \K is not connected. For and z $\notin$K, the convexity of K implies that there is line l_z through z such that K lies on one side of z. We call such a line "a separating line through z". We claim that all separating lines are parallel to each other. Indeed, if two separating lines l_z and l_{z1} through z and Z^I respectively, meet at a point z_O and z is any point outside K then lz" must meet either lz or lz". This implies that z"and z_o can be joined by a piecewise linear path. In particular, $\not\subset$ \K is connected, which is impossible by assumption. Now we may assume without loss of generality that 0 $\in$K and that all separating lines are parallel to the real axis. Suppose there are points iy_1 and iy_2 on the imaginary axis such that $y_1 \geq 0 \geq y_2$, $iy_1 \in k$, $iy_2 \in k$ and there are no points of K on the imaginary axis above iy_1 and below iy_2. Then it is easy to see that K is the closed strip bounded by the lines passing through iy_1 and iy_2 and parallel to the real axis. If such a y_1 or y_2 does not exist the $\not\subset$ \K is a half plane, which is a contradiction.

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